Paper Reference(s)

6684/01 Edexcel GCE

Statistics S2

Advanced Level

Thursday 24 May 2012 – Morning

Time: 1 hour 30 minutes

<u>Materials required for examination</u> Mathematical Formulae (Pink) Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S2), the paper reference (6684), your surname, other name and signature.

Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. This paper has 8 questions. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

P40106A This publication may only be reproduced in accordance with Edexcel Limited copyright policy ©2012 Edexcel Limited

- 1. A manufacturer produces sweets of length L mm where L has a continuous unifrom distribution with range [15, 30].
 - (*a*) Find the probability that a randomly selected sweet has length greater than 24 mm.

These sweets are randomly packed in bags of 20 sweets.

- (*b*) Find the probability that a randomly selected bag will contain at least 8 sweets with length greater than 24 mm.
- (c) Find the probability that 2 randomly selected bags will both contain at least 8 sweets with length greater than 24 mm.
 - (2)

(3)

(2)

2. A test statistic has a distribution B(25, p).

Given that

$$H_0: p = 0.5, H_1: p \neq 0.5,$$

- (*a*) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.
- (b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)

(3)

3. (*a*) Write down the two conditions needed to approximate the binomial distribution by the Poisson distribution.

(2)

A machine which manufactures bolts is known to produce 3% defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts are defective.

(d) Using a suitable approximation, test at the 5% level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.

(7)

- 4. The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.
 - (a) Find the probability that in the next four weeks the estate agent sells
 - (i) exactly 3 houses,
 - (ii) more than 5 houses.

(5)

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of those 4 week periods there are exactly nine periods in which more than 5 houses are sold.

(3)

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

(6)

5. The queuing time, X minutes, of a customer at a till of a supermarket has probability density function

$$f(x) = \begin{cases} \frac{3}{32}x(k-x) & 0 \le x \le k, \\ 0 & \text{otherwise.} \end{cases}$$

(<i>a</i>)	Show that the value of k is 4.	
(b)	Write down the value of $F(X)$	(4)
(0)	while down the value of $L(X)$.	(1)
(<i>c</i>)	Calculate Var (X).	(4)
		(-)
(<i>d</i>)	Find the probability that a randomly chosen customer's queuing time will differ from mean by at least half a minute.	the

6. A bag contains a large number of balls.

65% are numbered 1

35% are numbered 2

A random sample of 3 balls is taken from the bag.

Find the sampling distribution for the range of the numbers on the 3 selected balls.

(6)

(3)

7. The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} \frac{x^2}{45} & 0 \le x \le 3, \\ \frac{1}{5} & 3 < x < 4, \\ \frac{1}{3} - \frac{x}{30} & 4 \le x \le 10, \\ 0 & \text{otherwise.} \end{cases}$$

(<i>u</i>) Sketch $I(x)$ for $0 \le x \le 10$.	(4)
(<i>b</i>) Find the cumulative distribution function $F(x)$ for all values of <i>x</i> .	(8)
(c) Find $P(X \le 8)$.	(2)

8. In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

- (a) Find the probability that
 - (i) exactly 6 ask for water with their meal,
 - (ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of *n* such that

$$\mathbf{P}(X \le n) \ge 0.9,$$

where the random variable X represents the number of these customers who ask for water.

(3)

TOTAL FOR PAPER: 75 MARKS

Summer 2012 6684 Statistics S2 Mark Scheme

Question Number	Scheme	Mark	s
1(a)	$P(L>24) = \frac{1}{15} \times 6$ = $\frac{2}{5}$ or 0.4 oe	M1 A1	(2)
(b)	Let X represent the number of sweets with $L > 24$		
	<i>X</i> ~B(20, 0.4)	M1	
	$P(X \ge 8) = 1 - P(X \le 7)$	M1dep	
	= 1 - 0.4159		
	= 0.5841 awrt 0.584	A1	
			(3)
(c)	P(both $X \ge 8$) = (0.5841) ²	M1	
	= 0.341	A1 ft	
			(2)
		Tot	tal 7
	notes		
1(a)	M1 $\frac{1}{15}$ × (6 or 5.5 or 6.5 or (30 – 24)) or 1 - $\frac{1}{15}$ ((24 – 15) or (23.5 – 15) or (24.5 – 15)))	
(b)	M1 using B(20, "their (a))		
	M1 dependent on 1° M1. Writing or use of $1 - P(X \le 7)$		
	NB Use of normal/normal approximation/ Poisson/uniform gets M0 M0 A0		
(c)	M1 (their(b)) ² or $(0.58)^2$ or $(0.5841)^2$ or $(0.584)^2$		
	A1ft –either awrt 0.34 or follow through their answer to part (b) must be to 2sf or better.		
	Note you will have to check this.		

Question Number	Scheme	Marks
2.(a)	$X \sim B(25,0.5)$ may be implied by calculations in part a or b	M1
	$P(X \le 7) = 0.0216$	
	$P(X \ge 18) = 0.0216$	
	$\operatorname{CR} X \le 7; \cup X \ge 18$	A1,A1 (3)
(b)	$P(rejecting H_0) = 0.0216 + 0.0216$	M1
	= 0.0432 awrt 0.0432/0.0433	A1 (2)
		Total 5
	Notes	
2(a)	M1 - Using B(25,0.5) – may be implied by a correct critical region or by calculations in Note Just seeing either P($X \le 7$) or P($X \ge 18$) scores M1 A0 A0. You may need to check their probabilities in the tables for values other than 7 or 18. 1 st A1 – also allow $X < 8$ or [0,7] or $0 \le X \le 7$ or $0 \le X < 8$ oe e.g. [0, 8) or a full list DO NOT allow CRs given as P($X \le 7$) or 7 – 0 for the A mark. 2 nd A1 – also allow $X > 17$ or [18,25] or $18 \le X \le 25$ or $17 < X \le 25$ oe e.g. (17, 25] or a full list DO NOT allow CRs given as P($X \ge 18$) or $18 - 25$ for the A mark. SC $7 \ge X \ge 18$ gains M1 A1 A0.	part a or b
(b)	M1 – adding their two critical regions' probabilities together or may be awarded for awrt 0.0432 If they add their critical regions' probabilities and then go on and get a different probability as their answer then it is M0A0 e.g. $0.0216 + 0.0216 = 0.0432$ then $0.05 - 0.0432 = 0.0068$ gets M0 A0 e.g. $0.0216 + 0.0216 = 0.0432$ < 0.05 reject H ₀ gets M1 A1 e.g. $0.0216 + 0.0216 = 0.0432$ so probability of rejecting H ₀ is $1 - 0.0432 = 0.9568$ gets M0 A0	

Question Number		Scheme		Marks
3(a)	n - large/	/high/big/ <i>n</i> >50		B1
	p-small	/close to $0 / p < 0.2$		B1 (2)
(b)	$H_0: p = 0$ Po(6) P(X \ge 12	0.03 $H_1: p > 0.03$) = 1- P(X ≤ 11) or P(= 1 - 0.9799 P($(X \le 10) = 0.9574$ $X \ge 11) = 0.0426$	B1,B1 B1 M1
		= 0.0201 CR	$X \ge 11$	A1
	$(0.0201 < Reject H_0$ There is e	< 0.05) or Significant or 12 lies in the Critical reg evidence that the proportion of defective bo	ion. olts has increased.	M1 dep. A1 ft (7) Total 9
(b)	Notes 1^{st} B1 for H ₀ : $p = 0.03$ 2^{nd} B1 for H ₁ : $p > 0.03$ SC If both hypotheses are correct but a different letter to p is used they get B1 B0 Also allow B1 B0 for H ₀ : $\lambda = 6$ and H ₁ : $\lambda > 6$ B1 writing or using Po(6) <u>One tail</u> 1^{st} M1 for writing or using 1 - P($X \le 11$) or giving P($X \le 10$) = 0.9574 or giving P($X \ge 11$) = 0.0426. May be implied by correct CR or probability = 0.0201 1^{st} A1 for 0.0201 or CR $X \ge 11/X > 10$. NB P($X \le 11$) = 0.9799 on its own scores M1A1 2^{nd} M1 dependent on the 1^{st} M1 being awarded. For a correct statement based on the table below. Do not allow non-contextual conflicting statements eg "significant" and "accept H ₀ ". Ignore comparisons . 2^{nd} A1 ft for a correct contextualised statement. NB A correct contextual statement on its own scores			
	MIAI.	0.05 < <i>p</i> < 0.95	p < 0.05 or $p > 0.95$	
	2 nd M1	not significant/ accept H ₀ / Not in CR	significant/ reject H ₀ / In CR	
	2 nd A1	The proportion/number/amount/percentage oe of defective bolts has not increased/is not higher/oe	The proportion/number/amount/perc oe of defective bolts has increased/is h	<u>entage</u> iigher/oe
	Two tail 1^{st} M1 for 1^{st} M1 for May be in 1^{st} A1 for 2^{nd} M1 callow non- 2^{nd} A1 ftM1A1.	or writing or using 1 - P($X \le 11$) or giving P(X applied by correct CR or probability = 0.0201 or 0.0201 or CR $X \ge 12/X > 11$. NB P($X \le 11$ dependent on the 1 st M1 being awarded. For a -contextual conflicting statements eg "significa- for a correct contextualised statement. NB A co	$X \ge 12$ = 0.0201 or giving P($X \le 11$) =) = 0.9799 on its own scores M1A1 correct statement based on the table b ant" and "accept H ₀ ". Ignore compar orrect contextual statement on its own	= 0.9799. elow. Do not isons . scores
		0.025 < <i>p</i> < 0.975	<i>p</i> < 0.025 or <i>p</i> > 0.975	
	2^{nd} M1	not significant/ accept H ₀ / Not in CR	significant/ reject H ₀ / In CR	
	2 nd A1	The proportion/number/amount/percentage <u>oe</u> of <u>defective bolts</u> has <u>not increased/is not</u> higher/oe	The proportion/number/amount/perc oe of defective bolts has increased/is h	<u>entage</u> igher/oe
	Use of N	(6,5.82) May get B1 B1 B0 M1 (must use	11.5)A0 M1dep A1 ft	

Question Number	Scheme	Marks	
4(a)	Let <i>X</i> be the random variable the number of houses sold.		
	<i>X</i> ~Po(8)	B1	
(i)	$P(X \le 3) - P(X \le 2) = 0.0424 - 0.0138$ or $\frac{e^{-8}8^3}{3!}$	M1	
	- 0.0280 awit 0.0280		
(ii)	$P(X > 5) = 1 - P(X \le 5)$ = 1 - 0.1912	M1	
	= 0.8088 awrt 0.809	A1 (5)	
(b)	Let <i>Y</i> be the random variable = the number of periods where more than 5 houses are sold		
	$Y \sim B(12, 0.8088)$	M1	
	$P(Y=9) = (0.8088)^9 (1 - 0.8088)^3 \overline{93!}$	M1	
	= 0.228 awrt 0.228	A1 (3)	
(c)	N(20,20)	M1A1	
	$P(X > 25) = 1 - P\left(Z \le \frac{25.5 - 20}{\sqrt{20}}\right)$	M1,M1,A1	
	$= 1 - P (Z \le 1.23)$ = 1 - 0.8907	A 1	
	= 0.1095 / 0.1094 awit 0.109	(6) Total 14	
<i>.</i>	Notes	1000011	
(a) (i)	1st B1 for writing or using Po(8) in either (1) or (1) $e^{-8}8^3$		
~ /	M1 writing or using P(X \le 3) - P(X \le 2) or $\frac{e - 3}{3!}$		
(ii)	M1 writing or using 1 - P($X \le 5$)		
(b)	M1 writing or attempting to use B(12,their (a(ii))) NB ft their a(ii) to at least 2sf $\frac{12!}{12!}$ (a(ii)) ⁹ (1- a(ii)) ³ allow ¹² C ₂ or ¹² C ₀ or 220 instead of $\frac{12!}{12!}$ NB ft their a(ii) to at		
	least 1sf but an expression must be seen (No use of tables)		
(c)	 1st M1 for writing or using a normal approximation 1st A1 for correct mean and sd (may be given if correct in standardisation formula) 2nd M1 Standardising using their mean and their sd and using [24.5, 25, 25.5, 26 or 26.5] and for finding 		
	correct area by doing $1 - P(Z \le \text{ their } 1.23^\circ)$ NB if they have not written down a mean and sd then they need to be correct in the standardisat this mark	tion to gain	
	3^{rd} M1 for attempting a continuity correction (26± 0.5)		
	2^{nd} A1 for $\pm \frac{25.5 - 20}{\sqrt{20}}$ or $\pm a \text{ wrt } 1.2 \text{ or better.}$		
	SC using P(X< 26.5/25.5) – P(X<25.5/24.5) can get M1A1 M0M1A0A0		

Question Number	Scheme	Marks
5(a)	at 2	
J(d)	$\int_{0}^{x} \frac{3}{32} x(k-x) = 1$	M1
	$\frac{3}{32} \left[\frac{kx^2}{2} - \frac{x^3}{3} \right]^k = 1$	A1
	$\frac{3k^3}{3k^3} - \frac{3k^3}{3k^3} = 1$	M1 dep
	$ \begin{array}{r} 64 & 96 \\ 3k^3 - 2k^3 = 64 \end{array} $	
	$k^3 = 64$ k = 4	Alcso
b	[E(X) =]2	(4) B1
с	$E(X^2) = \int_0^4 \frac{3}{32} x^3 (4 - x)$	M1
	$=\left[\frac{3x^4}{32} - \frac{3x^5}{160}\right]_0^4$	
	$= \left[\frac{3 \times 4^4}{32} - \frac{3 \times 4^5}{160}\right]$	
	= 4.8 Var (X) = 4.8 - 4	A1 M1
	= 0.8	A1
d	$\int_{1.5}^{2.5} \frac{3}{32} x(4-x) = \left[\frac{3x^2}{16} - \frac{x^3}{32}\right]_{1.5}^{2.5} \text{or} \int_{0}^{1.5} \frac{3}{32} x(4-x) = \left[\frac{3x^2}{16} - \frac{x^3}{32}\right]_{0}^{1.5}$	(4) M1
	$= \frac{47}{128} = 0.3671875 \qquad \qquad = \frac{81}{256} = 0.31640625$	
	$1 - \frac{47}{128} = \frac{81}{128} \text{ awrt } 0.633 \qquad \qquad 2 \times \frac{81}{256} = \frac{81}{128} \text{ awrt } 0.633$	M1depA1
	Notes	Total 12
(a)	1 st M1 for an attempt to multiply out bracket and for attempting to integrate $f(x)$. Both $x^n \rightarrow x^{n+1}$ 1 st A1 for correct integration. Ignore limits for these two marks. Need $\frac{3}{2}\left(\frac{kx^2}{kx^2} - \frac{x^3}{kx^3}\right)$ or	
	$32\begin{pmatrix} 2 & 3 \end{pmatrix}^{32}$	
	2 nd M1 Dependent on the previous M mark being awarded. For correct use of correct limits and set equa $3 \begin{pmatrix} 4^3 & 4^3 \end{pmatrix}$	al to 1. No need
	to see 0 substituted in. For verifying they must have $\frac{3}{32}\left(\frac{4}{2}-\frac{4}{3}\right)$	
	2 nd A1 cso or for verifying $\frac{3}{32}\left(\frac{4^3}{2} - \frac{4^3}{3}\right) = 1$ or eg 3(4) ³ - 2(4) ³ = 64 and a correct comment "so $k =$	4"
(c)	1 st M1 attempt to multiply out bracket and attempting $\int x^2 f(x)$ Limits not needed. Both $x^n \rightarrow x^{n+1}$	
(d)	2^{nd} M1 for their E(X^2) – (their mean) ² 1 st M1 Multiply out brackets, attempting to integrate (both $x^n \rightarrow x^{n+1}$), with either limits (their(b) ± 0.5) of 0.5 and 0) Accept 2 sf for their limits.	or (their (b) –
	2^{nd} M1dep on gaining 1^{st} M1. 1 – (using limits (their(b) ± 0.5) or 2 × (using limits (their(b) – 0.5 at	nd 0)

Question Number	Scheme		Marks
6	Attempt to write down combinations	at least one seen	M1
	(1,1,1), (1,1,2) any order (1,2,2) any order, (2,2,2)	no extra combinations	A1
	Range 0 and 1	0 and 1 only	B1
	$[P(range = 0) =] (0.65)^{3} + (0.35)^{3}$ $= 0.3175 \text{ or } \frac{127}{400}$	either range	M1 A1cao
	$[P(range = 1) =] (0.35)^{2}(0.65) \times 3 + (0.65)^{2}(0.35) \times 3$ $= 0.6825 \text{ or } \frac{273}{400}$		Alcao
			(6)
			Total 6
	Notes		
	First M1 may be implied by either $(0.65)^3$ or $(0.35)^3$ or $(0.65)^2$ First A1 may be implied by $(0.65)^3$ and $(0.35)^3$ and $(0.65)^2$ No need for x3 2^{nd} M1 $(p)^3 + (1-p)^3$ or $(1-p)^2(p) \times 3 + (p)^2(1-p) \times 3$ A1 for 0.3175 cao or exact equivalent e.g $\frac{254}{800}$ A1 for 0.6825 cao or exact equivalent e.g $\frac{546}{800}$ NB These probabilities do not need to be associated with the) ² (0.35) or (0.35) ² (0.65) (0.35) and (0.35) ² (0.65) e correct range	



(a)	Notes 1^{st} B1 for a curve. It must start at (0, 0) and have the correct curvature.	
	2 nd B1 for a horizontal line that joins the first section of the graph (not by a dotted	
	line)	
	3 rd B1 for a straight line with negative gradient that joins the horizontal line and stops	
	on the positive x axis. 4^{th} B1 dependent on first 3 marks being gained. Fully correct graph with labels 0.2	
	3,4,10 in correct places	
(b)	For all the M marks, the attempt to integrate must have at least one $x^n \rightarrow x^{n+1}$	
	All A marks are for the correct expressions and ranges.	
	Do not penalise the use of \leq instead of $<$ and \geq instead of $>$.	
	$x t^2$	
	<u>1</u> st M1 for attempt to integrate $\int_0^x \frac{t}{45} dt$ ignore limits	
	2 nd M1	
	for attempt to integrate $\int_{-\infty}^{x} \frac{1}{-dt} dt$ + their F(3) using correct limits	
	for anomption integrate $\int_{3}^{3} \frac{1}{5} dx + dx + dx + dx$	
	or 1	
	for attempt to integrate $\int \frac{1}{5} dx + C$ and substituting in 3 and putting = to their F(3) or	
	substituting in 4 and putting = to their F(4) from their $4 \le x \le 10$ line	
	3 rd M1	
	for attempt to integrate $\int_{4}^{x} \frac{1}{3} - \frac{x}{30} dt$ + their F(4) using correct limits.	
	or	
	for attempt to integrate $\int \frac{1}{3} - \frac{x}{30} dt + C$ and substituting in 4 and putting = to their	
	F(4) or substituting in 10 and putting = 1	
(c)	M1 substituting 8 into the 4 th line of their cdf or $F(3) + F(4) - F(3) + F(8) - F(4)$ or	
	1 - $\int_{8}^{10} \frac{1}{3} - \frac{x}{30}$ (attempt to integrate needed) or use areas e.g $1 - \frac{1}{2} \times 2 \times \frac{1}{15}$ or $1 - \frac{1}{15}$	
	A1 14/15 awrt 0.933 from correct working.	
	NB If using $F(3) + F(4) - F(3) + F(8) - F(4)$ then $F(x)$ must be correct.	

Question Number	Scheme			Marks	
8(a)	Let <i>X</i> be the random variable the number of customers asking for water.				
(i)	$\frac{X \sim B(10,0.6)}{P(X = 6) = (0.6)^6 (0.4)^4 \frac{10!}{6!4!}} \qquad P(Y = 6)$	B(10,0.4) = 4) = (0.4) ⁴ (0.6) ⁶ $\frac{10!}{6!4!}$		B1 M1	
	= 0.2508 = 0.	2508	awrt 0.251	A1	
(ii)	$ \begin{array}{r} X \sim B(10, 0.6) \\ P(X < 9) = 1 - (P(X = 10) + P(X = 9)) \\ = 1 - (0.6)^{10} - (0.6)^9 (0.4)^1 \frac{10!}{9!!!} \\ = 0.9536 \end{array} $	$Y \sim B(10,0.4)$ $P(X < 9) = 1 - P(Y \le 1)$ $= 1 - 0.0464$ $= 0.9536$	awrt 0.954	M1 A1	
(b)	$\mathbf{Y} = \mathbf{R}(50, 0, 6)$	· · · · · ·		M1	(5)
(0)	$X \sim B(50, 0.6)$ $Y \sim B(50, 0.4)$ $P(X < n) \ge 0.9$ $P(Y > 50 - n) \ge 0.9$ or $P(Y \le 50 - n) \le 0.1$ $50 - n \le 15$ $n \ge 35$ n = 35	P(X < 34) = 0.8439 awrt 0.84 P(X < 35) = 0.9045 awrt 0.90	.4)4/0.905	M1 A1 Tota	(3) 1 8
(a)	Notes B1 writing or using $B(10,0.6) / B(10,0.4)$	4) in either part(i) or (ii)			
(i)	M1 $(0.6)^6 (1-0.6)^4 \frac{10!}{6!4!}$ Allow ${}^{10}C_6$ oe or writing or using P(X ≤ 6) - P(X ≤ 5) if using B(10,0.6) or P(X ≤ 4) - P(X ≤ 3) if using B(10,0.4) NB use of Poisson will gain MOA0				
(ii)	M1 writing or using $1 - (P(X = 10) + P(X = 10)) + P(Y \le 1)$ or $1 - P(Y \le 1)$ if using B(10,0.4)	(X = 9)) if using B(10,0.6)			
(b)	NB use of Poisson will gain M0A0 1^{st} M1 for writing or using either B(50,0.6) or B(50,0.4) 2^{nd} M1 P(Y > 50 - n) ≥ 0.9 or P(Y $\leq 50 - n$) ≤ 0.1 or P(X < 34) = awrt 0.844 or P(X < 34)				vrt
	SC use of normal. M1 M0 A0 for use of N(30,12) leading t	to an answer of 35			